**Counting – Notes**

n = Total number

r = Number of ways to arrange (number of spaces)

n is always greater than r

**Binomial expansion**:

(a+b)n = nCr a(n-r) br r = term number

Number of terms = (n+1)

(a+b)n = an + nC1 x a(n-1) x b + nC2 x a(n-2) x b2 + nC3 x a(n-3) x b3 . . .

r → ∞ (0 → n) n = order of polynomial

a → –∞ (n → 0)

b → ∞ (0 → n)

Examples:

(a+b)5 = 5C0 x a5 x b0 + 5C1 x a4 x b + 5C2 x a3 x b2 + 5C3 x a2 x b3 + 5C4 x a1 x b4 + 5C5 x a0 x b5

(2a–3b)3 = 3C0 x (2a)3 x b0 + 3C1 x (2a)2 x (–3b)1 + 3C2 x (2a)1 x (–3b)2 + 3C3 x (2a)0 x (–3b)3

**Permutations**: Arranging

nPr = $\frac{n!}{\left(n-r\right)!}$

**Combinations**: Choosing

( $\begin{matrix}n\\r\end{matrix}$ ) = nCr = $\frac{n!}{\left(n-r\right)! r!}$

**Pascal’s triangle**:

1

1C0 1C1

2C0 2C1 2C2

3C0 3C1 3C2 3C3

4C0 4C1 4C2 4C3 4C4

5C0 5C1 5C2 5C3 5C4 5C5

Raw totals:

Total:

1 = 20

2 = 21

4 = 22

8 = 23

16 = 24

32 = 25

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

**Rules**:

In (a+b)n the axis of symmetry is when a and b are at the same power (even orders) and when a and b “interchange” between the middle of Pascal’s triangle (odd orders).

The top numbers for combinations **must** add up to the grand total.

The bottom numbers **must** add up to the number of the things being chosen.

**Determine the value of integer r (where r ≥ 0):**

$( \begin{matrix}r\\0\end{matrix} )$ **+** $( \begin{matrix}r\\1\end{matrix} )$ **+** $( \begin{matrix}r\\2\end{matrix} )$ **+** $( \begin{matrix}r\\3\end{matrix} )$ **+** $( \begin{matrix}r\\4\end{matrix} )$ **= 2r**

r = Bottom number of last term = 4.

**Consider the expansion for ( x2 –** $\frac{2}{x}$ **)12 in descending powers of x.**

**[a] Find a mathematical expression for the coefficient of the term in** $\frac{1}{x^{12}}$**.**

nCr x (x2)n-r x (– $\frac{2}{x}$)r r = 12 because $\frac{1}{x^{12}}$ and n = 12 because the order is 12

12C12 x (x2)12-12 x (– $\frac{2}{x}$)12

1 x x 1 x (– $\frac{2}{x}$)12 = $\frac{4096}{x^{12}}$ = 4096 x $\frac{1}{x^{12}}$ → coefficient = 4096

**[b] Find a mathematical expression for the term independent of x.**

nCr x (x2)n-r x (– $\frac{2}{x}$)r r = 12 because $\frac{1}{x^{12}}$ and n = 12 because the order is 12

12Cr x (x2)12-r x ($-\frac{2}{x}$)r → x24-2r x ($-\frac{x}{2}$)r → 24 – 2r = r → 24 = 3r → r = 8

12C8 x (x2)4 x – ($\frac{2}{x}$)8 = 12C8 x (–2)8

**A committee of 9 people is to be selected from 10 Labour, 8 Liberal and 5 Green politicians. Write mathematical expressions for the number of different ways the committee can be selected if:**

**[a] The Liberal representatives are in the majority.**

8, 1 7, 2 6, 3 5, 4

8C8 x 15C1 + 8C7 x 15C2 + 8C6 x 15C3 + 8C5 x 15C4

**[b] The Labour husband and wife pair, Alex and Alice, can’t be in the same committee.**

23C9 – 2C2 x 21C7

[Combination with one of the pair]

2C1 x 21C8

**Consider the digits 0 to 9 inclusive and all the letters of the alphabet. 10 characters consisting of digits and letters are chosen. Determine the number of ways of choosing:**

**[a] All the even numbers and all the vowels.**

5C5 x 5C5 x 26C0 = 1

**[b] Any 6 digits and any 4 letters.**

10C6 x 26C4 = 3139500

**[c] Exactly 4 vowels.**

5C4 x 31C6 = 3681405

**[d] At least 4 odd digits.**

4, 6 5, 5

5C4 x 31C6 + 5C5 x 31C5 = 3851316

**[e] 4 vowels and 4 odd digits.**

5C4 x 5C4 x 26C2 = 8125

**[f] 4 vowels or 4 odd digits.**

5C4 x 31C6 + 5C4 x 31C6 – 5C4 x 5C4 x 26C2 = 7354685

**A selection of 3 people are to be chosen from a group of 6 people. How many selections are possible if the youngest or oldest is included but not both?**

2C1 x 4C2 = 12

**A selection of 4 people are to be chosen from a group of 7 people. How many selections are possible if the youngest or oldest is included but not both?**

5C3 x 2C1 = 20

**Harry’s organisation wants to form a partnership with a charity. After a day of speaking with representatives from different charities, he looks in his wallet and sees that he has 4 identical business cards from one charity and one business card from each of the other 7 charities he spoke with. If he wants to select 5 cards to keep, no two from the same charity, how many different selections can he make?**

There’s 4 of the same cards and 7 other different cards. He takes one of the 4 of the same cards and then the other 7 and so he has 8 total cards.

1C0 x 7C5 + 1C1 x 7C4 = 56

**In how many ways can 5 people be chosen from a group of 15 people if:**

**[a] The eldest is included?**

14C4 x 1C1 = 1001

**[b] The eldest isn’t included?**

14C5 = 2002

**[c] The eldest and youngest people are to be included?**

13C3 x 2C2 = 286